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On Under Water Explosions—A Comparative Study

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Abstract. Propagation and attenuation of spherical shock waves in water is studied theoretically by using Whitham's method and Energy Hypothesis method. Results are compared with the experimental data and it is found that attenuation predicted by Energy Hypothesis is quite agreeable with that obtained experimentally, where as that by Whitham's method gives higher values. A relation between two different forms of equations of state, generally reported in the literature, is established.

1. Introduction

Due to its applications in Naval Warfare as well as in Engineering problems, propagation and attenuation of shock waves in water is of immense importance. The works of Kirkwood and Bethe[1], Brinkeley and Kirkwood[2], Penny and Dasgupta[3] are worth mentioning. Details of these works are given in the classical book on the subject by R. H. Cole[4].

Later propagation of shock waves in water was studied theoretically using Whitham's method[5,6] and Energy Hypothesis[7,8]. In these papers, Tait's Equation of state for water was used. Singh[9] *et al.* found attenuation of spherical shock waves in ordinary water experimentally and compared the results with those predicted by the Energy Hypothesis[7,8]. It was found that shock velocity predicted by Energy Hypothesis is little higher than that obtained experimentally. In the above mentioned paper the equation of state was taken to be

$$U = a + bu_2$$

where a, b are constants of water and U, u_2 are shock velocity and particle velocity in water respectively.

In the present paper we have discussed different types of equations of state for water. In section 2, a relation between two types of equations of state is established. Equations of state of water reported by different authors is also discussed. In section 3, attenuation of spherical shock wave is found by Whitham's method of characteristics and is compared with that obtained by Energy hypothesis. The results of two theories are compared with experimental data[9].

It is concluded that attenuation obtained by Whitham's method of characteristics gives higher values to that obtained from experimental data where as energy Hypothesis give quite agreeable results. Kamel *et al.*[10], also compared, shock attenuation in water by Whitham's method with the method of shock trajectory. His results are also similar to our results. Incidently, this also proves that shock attenuation by the use of Energy Hypothesis, gives same result as that given by Kamel *et al.*[10]. We have used Tait's equation of state for water. Results are compared with few available equations of state.

2. Equation of State of Water

Generally two types of equations of state for water and even for metals are reported in literature. Cole[4] has reported Tait's equation of state for saline water which holds upto 50 Kilobars. Similar type of equation for ordinary water is reported in the literature[11,12].

Let us take Tait's form of equation of state for water

$$p = A\left[\left(\frac{\rho}{\rho_1}\right)^{n} \quad \right] \tag{1}$$

Where p is pressure, ρ is density and ρ_1 is density at atmospheric pressure. A and n are constants of water. Actually A is not a constant but a function of temperature. But for all practical purposes, we can take it as a constant, as the variations in A with temperature are small[10,13]. Jump condition across the shock front using Eqn. (1) are

$$p_2 - p_1 = A\,[\delta^n - 1] \tag{2}$$

$$u_2 = \left[\frac{A}{\rho_1}(\delta^n - 1)\left(\frac{\delta - 1}{\delta}\right)\right]^{1/2} \tag{3}$$

$$U = \left[\frac{A}{\rho_1}\,\frac{\delta(\delta^n - 1)}{(\delta - 1)}\right]^{1/2} \tag{4}$$

$$\delta = \left(\frac{\rho_2}{\rho_1}\right) \tag{4a}$$

where symbols have usual meaning[7]

A different form of equation of state of water is given as

$$U = a + bu_2 + cu_2^2 \tag{5}$$

where a, b, c are constants of water. Using Eqn. (5), the jump conditions across the shock front can easily be written as

$$p_2 \quad p_1 = \frac{\rho_1\delta(\delta - b(\delta - 1))^2}{2c^2(\delta - 1)^3}\left\{1 \quad \frac{2ac(\delta - 1)^2}{(\delta - b(\delta - 1))^2} \quad D\right\} \tag{6}$$

$$U \quad \frac{(\delta - b(\delta - 1))\,\delta}{2c(\delta - 1)^2}\left\{1 - D\right\} \tag{7}$$

$$u_2 = \frac{(\delta - b(\delta - 1))}{2c(\delta - 1)} (1 - D) \tag{8}$$

$$D = \left\{1 - \frac{4ac(\delta - 1)^2}{(\delta - b(\delta - 1))^2}\right\}^{1/2}$$

In Eqns. (5) to (8), c is quite small as compared to a and b. If we neglect c as compared to a and b, Eqns. (6) to (8) reduce to,

$$p_2 - p_1 = \frac{\rho_1 a^2 \delta(\delta - 1)}{\{\delta - b(\delta - 1)\}^2} \tag{9}$$

$$U = \frac{a\delta}{\{\delta - b(\delta - 1)\}}$$

$$u_2 = \frac{a(\delta - 1)}{\{\delta - b(\delta - 1)\}}$$

These equations are same as given by Singh *et al.*[9].

Eqns. (1) and (5) can easily be related. In the case of water, δ is always less than 2 for available conventional explosives, we can write

$$\delta = 1 + \Delta \tag{12}$$

where $\Delta < 1$. Using Eqn. (12) in Eqns. (2) and (6) and expanding one gets after neglecting higher order terms,

$$p_2 - p_1 = An\Delta\left[1 + \frac{n-1}{2}\Delta + \frac{(n-1)(n-2)}{6}\Delta^2 + \dots\right]$$

$$p_2 - p_1 = \rho_1 a^2 \Delta\,[1 + (2b - 1)\Delta + \{3(b-1)^2 + 2(b - 1) + 2ac\}\Delta^2 + \dots]$$

On comparing terms of equal powers of Δ in Eqns. (13) and (14), one gets

$$a = \sqrt{\frac{An}{\rho_1}}, \quad b = \frac{n+1}{4}$$

$$c = -\frac{1}{96}\sqrt{\frac{\rho_1}{An}}(n + 1)(n - 7)$$

$$- \frac{b}{6a}(b - 2)$$

which shows that c is a function of a and b only and is not independent. When $b = 2$, i.e. $n = 7$, we get a linear relation between U and u_2.

Equations (15) to (17) relate two types of equations of states. Thus when one equation of state is given, second one can easily be derived.

3. Propagation and Attenuation of Shock Waves in Water

Present author has studied propagation of spherical shock waves in water using Energy Hypothesis[7,8,9] and Whitham's method of characteristics[5,6]. In these papers both equations of states, as discussed in Section 2 were used. Variation of shock strength δ for the case of Energy Hypothesis[7,8] is given by

$$\frac{R}{2}\frac{d\delta}{dR} = -\frac{3}{2}\frac{(\delta - 1)(\delta^n - 1)}{n\delta^{n-1}(\delta - 1) + (\delta^n - 1)} \tag{18}$$

and by Whitham's methods of characteristics[6] is given by

$$\frac{R}{2}\frac{d\delta}{dR} = -\delta^2 K(\delta) \tag{19}$$

$$K(\delta)^{-1} = \tfrac{1}{2}\delta\left[1 + \left\{\frac{n\delta^n}{(\delta^n - 1)(\delta - 1)}\right\}^{1/2}\right]\left[1 + \tfrac{1}{2}\sqrt{\frac{n\delta^n(\delta - 1)}{(\delta^n - 1)}} + \tfrac{1}{2}\sqrt{\frac{(\delta^n - 1)}{n\delta^n(\delta - 1)}}\right] \tag{20}$$

In Eqns. (18) and (19), R is the distance of spherical shock from the centre of explosion and equation of state (1) and jump conditions (2)–(4) are used to derive these equations.

Equations (18) and (19) are integrated by using Ranga Kutta method of fourth order, with the help of DEC–20 computer. It is assumed that spherical shock is produced by detonating a spherical charge of *RDX/TNT*, 60 : 40, in the water tank[9]. Initial value of δ, just at the water-explosive boundary is found by mismatch method of shock impedence[15]. Variation of shock velocity versus radius R is shown in Fig. 1, by both the methods. Circles are experimental points[9]. In Fig. 2, we have shown the variations of shock velocity U versus particle velocity u_2. This is second degree curve of the type assumed in Eqn. (5). We have fitted a second degree curve in U and u_2 by the method of least squares. Values of A and n are taken[11,12] to be 2.94 kilobar and 7.25. Values of a, b, c from the fitted curve are :

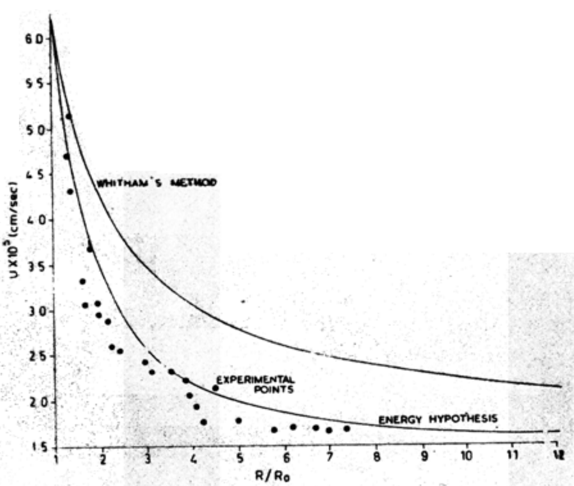

Figure 1. Variation of shock velocity *vs* shock radius by two different theories and comparison with the experimental data.

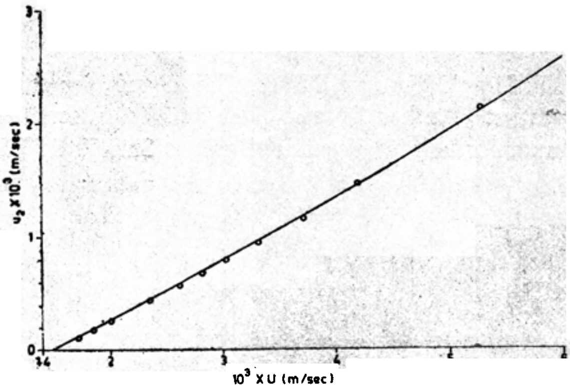

Figure 2. Variation of shock velocity u vs particle velocity u_2.

$$a = 1.46597 \text{ km/sec}, \quad b = 2.0122, \quad \text{and} \quad c = -0.08714.$$

Values of a, b, c from a Eqns. (12) to (17) are :

$$a = 1.4586 \text{ km/s}, \quad b = 2.0625, \quad \text{and} \quad c = -.01473.$$

Walsh and Rice[16] have found variation of shock parameters upto 400 kilobars of pressure. Values of a, b, c by fitting a second degree curve in their data are :

$$a = 1.5745, \quad b = 1.9462, \quad c = -.09726.$$

Values of A and n, corresponding to these values of a and b are :

$$A = 3.6617 \text{ K. bar}, \quad n = 6.785.$$

Cook[14] has given the values $a = 1.51$, $b = 1.85$, corresponding to which $A = 3.62714$ kb, $n = 6.30$.

It can be seen that values of a, b, c given by different authors, differ slightly. This deviation can be probably due to the different types of impurities of natural water.

4. Conclusion

It is concluded from the discussions of previous sections, that energy hypothesis gives better prediction of shock waves propagation in water as compared to that by Whitham's method. Similar observation is also reported by Kamel[10] *et al.* in their work.

Equations of state of water given by various authors is compared and it can be seen that so far this equation is not standardised. This is due to the fact that water constituents vary from place to place. Equations given by Walsh and Rice[16] can be taken as standard for all practical purposes. Relation between Tait's equation of state and shock-particle velocity relation, established in section 2, also holds for metals. But these relations may show deviations when the shock is very strong i.e. when $\Delta \approx 1.0$.

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